11 - More propositional logic

Example. "If Dan scores ≥ 93, then Dan gets A." is true. Two cases:

(a) "Dan scores <93" is also true.
What can you conclude?

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

(b) "Dan does not get A" is also true.
What can you conclude?

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

We can understand $\neg(p \Rightarrow q)$ via its truth table:

$$\neg(p \Rightarrow q) \text{ true } \equiv \text{row 2 true}$$
$$\equiv p \text{ true and } \neg q \text{ true}$$
$$\text{So: } \neg(p \Rightarrow q) \equiv p \wedge \neg q$$

p	q	$\neg(p \Rightarrow q)$
T	T	F
T	F	T
F	T	F
F	F	F

This procedure finds the **disjunctive normal form (DNF)** of a proposition.

Example. Put $p \oplus q$ into DNF.

$$p \oplus q \text{ true } \equiv \text{row 2 true OR row 3 true}$$
$$\equiv (p \text{ true AND } \neg q \text{ true}) \text{ OR } (\neg p \text{ true AND } q \text{ true})$$
$$\text{So: } p \oplus q \equiv (p \wedge \neg q) \vee (\neg p \wedge q)$$

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Proposition p is a **tautology** if $p \equiv T$, and a **contradiction** if $p \equiv F$.

Example. Show $p \vee \neg p$ is a tautology.

p	$\neg p$	$p \vee \neg p$
T	F	T
F	T	T

since the truth table always output True,
$p \vee \neg p$ is a tautology.

Optional Homework due April 1 or 2.
Show your work. Answer without work receives no credit.

1. Write the DNF of $p \Leftrightarrow q$ which is defined as $(p \Rightarrow q) \wedge (q \Rightarrow p)$
2. Verify "modus ponen" $(p \Rightarrow q) \wedge p \Rightarrow q$ and "modus tollens" $(p \Rightarrow q) \wedge \neg q \Rightarrow \neg p$ are tautologies.
3. Consider the statement "If Bob eats tacos, then he drinks milk."
   (a) Write the converse of the statement.
   (b) Write the contrapositive of the statement.
   (c) The original statement is true, and Bob drinks milk. What can you conclude about his eating tacos?
   (d) The original statement is true and Bob drinks no milk. Can you conclude anything about his eating tacos?

Non-Homework Problems.

4. The statements "Jon eats pizza or drinks tea (possibly both)." and "If Jon eats pizza, then Jon drinks tea." are true. Is "If Jon does not eat pizza, then Jon drinks tea." also true? (Hint: write the truth table of the two given statements, and rule out the rows that cannot occur.)

12 - Predicate logic

A **predicate** P is a statement with variables where the variable are evaluated in some set U called the **domain of discourse**. We say P is a predicate over U.
Examples:

- isPrime(n) = "n is a prime number" over $U = \mathbb{N}$
  isPrime(5) = "5 is a prime number" is true
- isPowerOf(n,k) = "$n = k^i$ for some $i \in \mathbb{N}$" over $U = \mathbb{Z}^2$
  isPowerOf(8,2) = "$8 = 2^i$ for some $i \in \mathbb{N}$" is a true prop since $8 = 2^3$ and $i = 3 \in \mathbb{N}$
  isPowerOf(2,8) = "$2 = 8^i$ for some $i \in \mathbb{N}$" is a false prop since $8 = 2^{1/3}$ and $i = 1/3 \notin \mathbb{N}$

Let P be a predicate over U. Then

$\forall x \in U : P(x)$ = "For every x∈U, P(x) is true." (For every, for all)

$\exists x \in U : P(x)$ = "There exists x∈U such that P(x) is true." (There exists, for some, for at least one)

are defined to be propositions that satisfy:

$\forall x \in U : P(x)$ is true $\Leftrightarrow$ $P(x)$ is true for all $x \in U$
$\exists x \in U : P(x)$ is true $\Leftrightarrow$ $P(x)$ is true for at least one $x \in U$

We call $\forall$ and $\exists$ the **universal and existential quantifiers.**

Examples.

$\forall n \in \mathbb{N} : \text{isPrime}(n)$ = "For every natural number n, n is prime" is false: n=4 is not prime.

$\exists n \in \mathbb{N} : \text{isPrime}(n)$ = "There exists a natural number n such that n is prime" is true: n=5 is prime.

Examples. Formalize the following statements.

$$\text{isPrime}(n) \equiv (n \geq 2) \wedge [(d \mid n) \Rightarrow (d = n) \vee (d = 1)]$$

$$\text{isPowerOf}(n, k) \equiv \exists i \in \mathbb{N} : n = k^i$$